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An Analytic Approximation to Very High Specific Impulse and Specific Power Interplanetary Space Mission Analysis

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AN ANALYTIC APPROXIMATION TO VERY HIGH SPECIFIC IMPULSE & SPECIFIC POWER INTERPLANETARY SPACE MISSION ANALYSIS

Craig Hamilton Williams*

A simple, analytic approximation is derived to calculate trip time and performance for propulsion systems of very high specific impulse (50,000 to 200,000 seconds) and very high specific power (10 to 1000 kW/kg) for human interplanetary space missions. The approach assumed field-free space, constant thrust/constant specific power, and near straight line (radial) trajectories between the planets. Closed form, one dimensional equations of motion for two-burn rendezvous and four-burn round trip missions are derived as a function of specific impulse, specific power, and propellant mass ratio. The equations are coupled to an optimizing parameter that maximizes performance and minimizes trip time. Data generated for hypothetical one-way and round trip human missions to Jupiter were found to be within 1% and 6% accuracy of integrated solutions respectively; verifying that for these systems, credible analysis does not require computationally intensive numerical techniques.

INTRODUCTION

Through countless millennia, mankind has ventured to unknown worlds in the pursuit of wealth, knowledge, freedom, adventure, and international prestige. To date, human exploration of space has been primarily driven by the pursuit of international prestige. Because of the high cost and long trip times associated with human interplanetary exploration, such endeavors are expected to remain unattractive to any potential sponsor other than the governments of the most financially capable and technologically advanced nations. While it is difficult to imagine a low cost solution to human interplanetary travel, far-term technically feasible concepts do exist that could significantly reduce the expected long trip times. This could obviate larger and more complex spacecraft, higher operations costs, inordinately long return on investment times compared to commercial ventures, and the unappealing sacrifice of the spacefarer's personal time. Conventional chemical, and nearer-term electric and nuclear fission propulsion technologies would require multi-year round-trip times to even the closest planets. Although long duration missions have been the norm for unmanned probes, they represent a real hardship for human travel. More advanced propulsion technologies, however, could provide trip times comparable to some commercial terrestrial operations --- several weeks, perhaps even days. As D. Cole pointed out in his 1959 paper on minimum trip time travel to the planets, the evolution of terrestrial propulsion systems has been from the inexpensive "minimum energy" sailing vessels to the more costly "minimum time" aircraft[1]. Advanced space propulsion systems could catalyze future interplanetary travel predicated on the private, not public, sector market. High specific impulse(I_{sp})/high specific power (α) systems could accomplish this by traversing almost straight line trajectories between planets due to their expected ability to generate large accelerations. These propulsive technologies, unfortunately, are well beyond near-term feasibility and require solving some formidable technical challenges.

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Because of the technological immaturity and perceived remoteness of these advanced propulsion concepts, most human interplanetary mission studies have been predicated on trip time and propulsion system performance data generated by analysis tools designed to model either I_{sp}-limited systems (high thrust) or α-limited systems (low thrust). As a result, high I_{sp}/high α technologies are infrequently studied and therefore have few analytic tools designed to model them. Fortunately, due to the nature of their operation, simple analytical techniques can provide reasonably accurate estimates of their mission travel times and performance capabilities. These simple, first-order analytical methods can greatly assist in focussing development of requisite precursor technologies. Very high thrust propulsion systems could produce accelerations greater than the local acceleration due to solar gravity at Earth's orbit (0.6 E-3 g). The normally thought-of conics of minimum energy trajectories degenerate into straight line transfers at these acceleration levels. A "field-free space" approximation can then be invoked to greatly simplify the usually complex orbital mechanics. Gravity losses and optimum steering concerns can be neglected without introducing too much error, obviating the need for computationally intensive, numerically integrated solutions to support a proof of concept analysis. As a result, an analytic closed-form solution can be derived that calculates minimized trip time and maximized performance capability in a single equation (or at most two).

An extensive literature search surfaced three published approaches that solve only select portions of this problem for a constant thrust engine; though it is also worth noting that a few papers have also been written on the simpler to analyze, constant acceleration (variable thrust) device[1]. During the 1950's, H. Preston-Thomas of the National Research Council of Canada derived several one dimensional velocity equations[2]. Found just prior to the publication of this paper, Preston-Thomas' work contains trip time relations, but sparse derivations and few equations of motion made assessment of the approach difficult. Wolfgang Moeckel of the NASA Lewis Research Center developed a simple analytic approach using a one dimensional "flat solar system" model to calculate trip time using the quotient of velocity and distance during the 1960's[3]. High and low thrust systems were treated separately; the high thrust relations based on vehicle ΔV calculated by the classic rocket equation, the low thrust relations based on the time integral of acceleration squared equation ("J" parameter), modified to optimize engine-on time. Moeckel's approach had the advantage of extreme simplicity with reasonably good accuracy. However, the segregation of high and low thrust systems into separate equations did not illustrate the explicit dependence of trip time as an function of only distance, I_{sp}, and α in advanced very high I_{sp}/very high α systems.

Professor Dennis Shepherd of Cornell University developed an alternate approach in the 1960's[4]. Based in part on some earlier work by Ernst Stuhlinger, Shepherd's approach used a one dimensional, finite burn model (continuous thrusting with no coast phases). He derived a closed-form equation for distance traveled in field-free space by an accelerating rocket at constant thrust in terms of I_{sp}, α, and trip time. The classic rocket equation served as the starting point, later incorporating an optimizing parameter to maximize performance and minimize trip time. One advantage of Shepherd's finite burn approach was a more conservative representation of one dimensional motion. More important, however , was the explicit appearance of I_{sp} with α in a single equation optimized for payload and trip time, thus providing the analyst with a convenient way of evaluating the relative effect of distance, I_{sp} , and α on trip time for any mission distance. The main limitation was that the relation was valid for one-way flyby missions only, where the spacecraft would not rendezvous with the destination planet.

The problem solved in this paper is an extension of Shepherd's work. Two-burn "rendezvous" and four-burn "round trip" mission equations of motion are derived with an optimization condition imposed. Each leg of the trip consists of an acceleration burn from Earth's heliocentric position followed immediately by a deceleration burn (with no intervening coast period) which terminates at the heliocentric position of the destination planet (with zero radial velocity with respect to the sun). The transfers are assumed to originate and terminate outside the effective planetary gravity wells. These one dimensional straight line approximations along the heliocentric radius vector closely resemble numerically integrated solutions (Figure 1).

APPROACH

Review and Reformulation of the Solution to the One-Burn Flyby Problem

Shepherd's [4] approach will be briefly summarized. The basic rocket equation for a constant I_{sp} vehicle is:

$$\Delta V = c \ln\left(1-\frac{M_p}{M_i}\right)^{-1} = c \ln(1-\lambda)^{-1} \quad (1)$$

The mass balance relation is given by:

$$M_p + M_s = M_i - M_{pay} \quad (2)$$

Rearranging terms of the mass equation yields:

$$M_p\left(1 + \frac{M_s}{M_p}\right) = M_i\left(1 - \frac{M_{pay}}{M_i}\right) \quad (3)$$

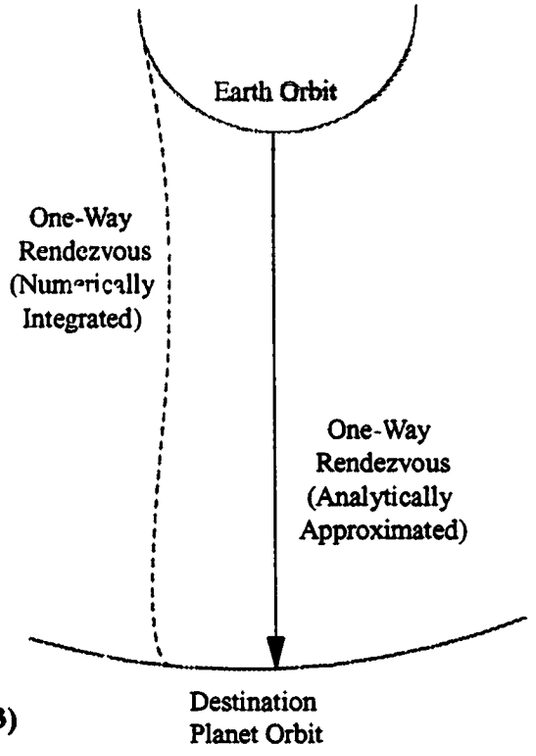

Figure 1: Continuously Thrusting Trajectories

Substituting for the propellant mass ratio into the rocket equation yields:

$$\Delta V = c \ln\left[\frac{1+\frac{M_s}{M_p}}{\frac{M_{pay}}{M_i}+\frac{M_s}{M_p}}\right] \quad (4)$$

From the definitions of jet power and specific power, a structure to propellant mass ratio can be defined:

$$P=\frac{\dot{M}_p c^2}{2g_c} \quad (5) \qquad \alpha=\frac{P}{M_s \eta} \quad (6) \qquad \textit{thus:} \quad \frac{M_s}{M_p} = \frac{c^2}{2g_c \eta \alpha T} \quad (7)$$

Shepherd introduced a parameter he called the ***characteristic velocity*** V_c, which upon substitution allows for a useful expression for the structure to propellant mass ratio:

$$V_c \equiv \sqrt{2 g_c \eta \alpha T} \qquad \textbf{(8)} \qquad \textit{thus:} \quad \frac{M_s}{M_p} = \left(\frac{c}{V_c}\right)^2 \qquad \textbf{(9)}$$

Substituting Eq. (9) into Eq. (4) yields a relation for payload ratio in terms of ΔV, exhaust and characteristic velocities:

$$\frac{M_{pay}}{M_i} = \frac{1 + \left(\frac{c}{V_c}\right)^2}{\exp^{\frac{\Delta V}{c}}} - \left(\frac{c}{V_c}\right)^2 \qquad \textbf{(10)}$$

Shepherd illustrated through a series of plots the implications of such an expression, primarily the existence of an optimal I_{sp} (in which the payload is maximized) for a particular mission ΔV. He demonstrated this by differentiating the above expression with respect to the parameter c/V_c and setting it equal to zero. The result is the optimization parameter:

$$\exp^{\frac{\Delta V}{c}} = \frac{1}{1-\lambda} = \frac{\Delta V}{2c}\left[1 + \left(\frac{V_c}{c}\right)^2\right] + 1 \qquad \textbf{(11)}$$

Shepherd then took the classical distance equation for a single burn, continuously accelerating rocket (neglecting gravity losses):

$$S(t) = V_0 t + \frac{cT}{\lambda}\left[\left(1 - \lambda\frac{t}{T}\right)\ln\left(1 - \lambda\frac{t}{T}\right) - \left(1 - \lambda\frac{t}{T}\right) + 1\right] \qquad \textbf{(12)}$$

and solved for total distance traveled in time T (assuming zero initial velocity):

$$S = \frac{cT}{\lambda}[(1-\lambda)\ln(1-\lambda) + \lambda] = cT\left[\frac{\ln\left(\exp^{\frac{-\Delta V}{c}}\right)}{\exp^{\frac{\Delta V}{c}} - 1} + 1\right] \qquad \textbf{(13)}$$

By substituting the optimization parameter Eq. (11) for the exponential term in the denominator of Eq. (13), we have:

$$\frac{S}{cT} = \frac{1-\left(\frac{c}{V_c}\right)^2}{1+\left(\frac{c}{V_c}\right)^2} \qquad \textbf{(14)}$$

At this point, Shepherd concluded with discussion of the implications of this acceleration-only distance equation.

What is of primary interest is to rewrite Shepherd's single burn flyby mission distance equation into a form where trip time is explicitly a function of distance, I_{sp}, and α. Expressing c as $g_c I_{sp}$ and recalling the definition of V_e, Eq.(14) can be rewritten as a quadratic in T:

$$T^2 - \left[\frac{S}{g_c I_{sp}} + \frac{g_c I_{sp}^2}{2\alpha\eta}\right] T - \frac{S I_{sp}}{2\alpha\eta} = 0 \qquad \textbf{(15)}$$

Solving for trip time via the quadratic formula:

$$T_{1,2} = \frac{1}{2}\left[\frac{S}{g_c I_{sp}} + \frac{g_c I_{sp}^2}{2\alpha\eta}\right] \pm \frac{1}{2}\sqrt{\left(\frac{g_c I_{sp}^2}{2\alpha\eta}\right)^2 + \frac{3 S I_{sp}}{\alpha\eta} + \left(\frac{S}{g_c I_{sp}}\right)^2} \qquad \textbf{(16)}$$

Where only the positive radical term is meaningful.

Shepherd demonstrated that c/V_e has limits of $\approx 0.505 <= c/V_e <= 1.0$, where the lower limit corresponds to a zero payload ratio and the upper limit corresponds to a payload ratio of unity. These limits are a manifestation of imposing the optimization parameter on the rocket equation. Since the payload ratio is a function of c/V_e and $\Delta V/c$, and $\Delta V/c$ is a function of only c/V_e, specifying c/V_e is equivalent to specifying a payload ratio. Since c/V_e is solely a function of T, I_{sp}, and α, payload ratio can replace one of these three variables. For example, it was convenient to express the limiting $\alpha = f(I_{sp}, S)$ for the zero payload ratio case:

$$\frac{c}{V_c} = \frac{g_c I_{sp}}{\sqrt{2 g_c \eta \alpha T}} \approx 0.505 \quad for \quad \frac{M_{pay}}{M_i} = 0 \qquad \textbf{(17)}$$

$$therefore \quad \alpha_{lim} = \frac{g_c I_{sp}^2}{2(0.505)^2 \eta T} \qquad \textbf{(18)}$$

Substituting Eq. (18) into Eq. (16) and selecting the meaningful root results in an equation for the limiting trip time:

$$T_{\lim} = 1.68\frac{S}{g_c I_{sp}} \qquad \textbf{(19)}$$

Thus the minimum trip time for a single burn flyby mission with negligible payload and optimal α varies linearly with distance and inversely with I_{sp}. The optimal α can then be calculated from Eq. (18).

Derivation of the Solution to the Two-Burn Rendezvous Problem

Unlike the one-burn flyby mission, the rendezvous mission would include a deceleration burn to reduce the sun-relative radial velocity to zero by the time of arrival at the destination planet. The rendezvous mission also presupposed the utilization of in situ resources at the destination planet to refuel the propulsion system, since the vehicle would carry only enough propellants for a one way trip. A modified rocket equation to model the deceleration burn is first needed. Shepherd's time dependent form of the constant mass flow/constant thrust rocket equation for the acceleration burn ($0 < t <= t_1$) was:

$$V_1(t) = V_0 - g_c I_{sp} \ln\left[1 - \lambda_1 \frac{t}{t_1}\right] \qquad \textbf{(20)}$$

Therefore, a new deceleration burn ($t_1 <= t < t_2$) velocity function can be modeled similarly as:

$$V_2(t) = V_1 + g_c I_{sp} \ln\left[1 - \lambda_2 \frac{t - t_1}{t_2 - t_1}\right] \qquad \textbf{(21)}$$

V_1 is the velocity at the turn around point. This point is where the vehicle terminates acceleration, rotates 180 degrees, and commences the deceleration burn. The logarithmic term, therefore, is a negative or "braking" ΔV. The distance and acceleration expressions for the deceleration phase are easily derivable. To integrate the logarithmic term, the following substitution is convenient:

$$let\ u \equiv 1 - \lambda_2\left(\frac{t - t_1}{t_2 - t_1}\right) \quad then\ \frac{du}{dt} = \frac{-\lambda_2}{t_2 - t_1} \qquad \textbf{(22)}$$

$$S_2(t) = V_1 \int_{t_1}^{t} dt + g_c I_{sp} \int_{t_1}^{t} \ln\left[1 - \lambda_2\left(\frac{t - t_1}{t_2 - t_1}\right)\right] dt = V_1(t - t_1) + g_c I_{sp} \frac{t_2 - t_1}{-\lambda_2} \int_{1}^{u} \ln u\ du \qquad \textbf{(23)}$$

Completing the integration results in an equation for deceleration distance as a function of time:

$$S_2(t) = V_1(t=t_1) - \frac{g_c I_{sp}(t_2-t_1)}{\lambda_2}\left[\left(1-\lambda_2\frac{t-t_1}{t_2-t_1}\right)\ln\left(1-\lambda_2\frac{t-t_1}{t_2-t_1}\right)-\left(1-\lambda_2\frac{t-t_1}{t_2-t_1}\right)+1\right] \quad (24)$$

Taking the time derivatives of the velocity equations result in the acceleration and deceleration equations as a function of time:

$$A_1(t) = \frac{dV_1(t)}{dt} = \frac{g_c I_{sp}\lambda_1}{t_1\left[1-\lambda_1\frac{t}{t_1}\right]} \quad (25)$$

$$A_2(t) = \frac{dV_2(t)}{dt} = \frac{-g_c I_{sp}\lambda_2}{(t_2-t_1)\left[1-\lambda_2\frac{t-t_1}{t_2-t_1}\right]} \quad (26)$$

A representative plot of the position, velocity, and acceleration expressions for an example mission are illustrated in figure 2.

A total distance equation can be formed and the variables S_1, S_2, V_1, t_1, t_2, λ_1, and λ_2 replaced with more convenient variables by inserting known relations and constraints. The total distance traveled is merely:

$$S = S_1 + S_2 \quad (27)$$

The magnitude of the velocity at end of the acceleration phase is:

$$V_1 = -g_c I_{sp}\ln(1-\lambda_1) \quad (28)$$

It is reasonable to assume that the magnitude of the change of velocities will be equal:

$$-g_c I_{sp}\ln\left[1-\lambda_1\left(\frac{t_1}{t_1}\right)\right] = -g_c I_{sp}\ln\left[1-\lambda_2\left(\frac{t_2-t_1}{t_2-t_1}\right)\right] \quad (29)$$

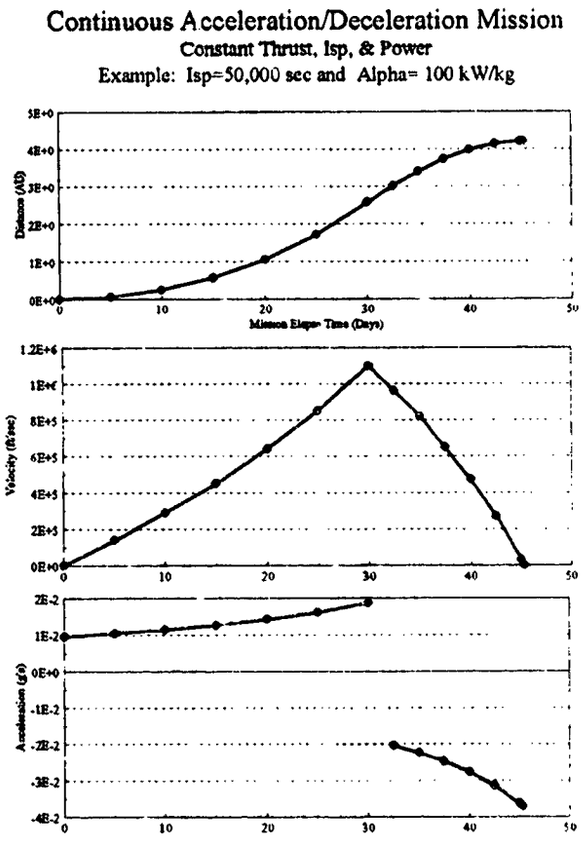

Figure 2: Position, Velocity, & Acceleration

Thus, the stage propellant ratios are equal.

$$\lambda_1 = \frac{M_{p_1}}{M_i} = \lambda_2 = \frac{M_{p_2}}{M_i - M_{p_1}} = \lambda \tag{30}$$

Adding the two distance expressions (Eqs. (12) and (24), evaluated at their respective total burn times) and substituting Eqs. (28) and (30) results in an equation for total distance traveled:

$$S = g_c I_{sp} t_1 \left[\left(\frac{1-\lambda}{\lambda} \right) \ln(1-\lambda) + 1 \right] - g_c I_{sp} (t_2 - t_1) \ln(1-\lambda) - g_c I_{sp} (t_2 - t_1) \left[\left(\frac{1-\lambda}{\lambda} \right) \ln(1-\lambda) + 1 \right] \tag{31}$$

The total distance traveled, S, is the straight line distance between the orbits of the two planets. It may be set to the difference between the radii (shortest trip time) or any other distance up to the opposition of the planets.

Note that the deceleration phase propellant ratio with respect to the initial mass can be written:

$$\lambda_2 = \frac{M_{p_2}}{M_i - M_{p_1}} = \frac{\frac{M_{p_2}}{M_i}}{1 - \frac{M_{p_1}}{M_i}} = \frac{\frac{M_{p_2}}{M_i}}{1-\lambda_1} \quad thus \quad \frac{M_{p_2}}{M_i} = \lambda(1-\lambda) \tag{32}$$

A relationship can be found relating the two burn times by noting that the mass flow rates are equal and that the total mission time is merely the sum of the two burn phases:

$$\frac{M_{p_1}}{t_1} = \frac{M_{p_2}}{t_2 - t_1} \quad thus \quad \frac{t_1}{t_2 - t_1} = \frac{\frac{M_{p_1}}{M_i}}{\frac{M_{p_2}}{M_i}} = \frac{1}{1-\lambda} \tag{33}$$

After replacing t_1 in Eq. (31) with Eq. (33), and noting that t_2 is the total trip time T, the logarithmic terms cancel and the distance equation reduces to:

$$S = \frac{g_c I_{sp} T \lambda}{2-\lambda} \tag{34}$$

This is quite different from the acceleration-only distance Eq. (13), which is a logarithmic function of λ. The implication is that attempting to substitute the optimization parameter Eq. (11) (with its exponential function of ΔV/c) into the distance equation, as was done in the one-burn flyby case, will not result in a simple function of distance, I_{sp}, and α (since there will not be a clean cancellation of the intermediate variable ΔV/c). The distance and optimization equations for the two-burn rendezvous case (as well as the four-burn round trip case as we shall see) are therefore coupled by the dependent variable λ and must be solved by iteration

It was found to be easiest to have λ serve as the iteration parameter. Remember that the λ used above was defined as a stage propellant ratio. It must first be replaced with the equivalent expression for the total propellant ratio, which is implicit in the optimizing parameter equation

$$1-\exp^{\frac{-\Delta V}{g_c I_{sp}}} = \lambda_{total} = \frac{M_{p_1}+M_{p_2}}{M_i} = \frac{M_{p_1}}{M_i}+\frac{M_{p_2}}{M_i} = \lambda_{stage}+\lambda_{stage}(1-\lambda_{stage}) \tag{35}$$

This quadratic in λ_{stage} has a solution of :

$$\lambda_{stage} = 1 \pm \sqrt{1-\lambda_{total}} \tag{36}$$

For the cases of interest here, where ≈0.505<= c/V_c<=1.0, only the negative root term is meaningful. Using this expression to replace λ_{stage} with λ_{total} in Eq. (34) yields:

$$S = \frac{g_c I_{sp} T(1-\sqrt{1-\lambda_{total}})}{(1+\sqrt{1-\lambda_{total}})} \quad or \quad \lambda_{total} = 1-\left(1-\frac{2S}{g_c I_{sp} T+S}\right)^2 \tag{37}$$

The optimizing parameter Eq. (11) can be expressed as a function of λ_{total}, I_{sp}, and α by recalling the definition of λ from the rocket equation and the definition of c/V_c as a function of I_{sp}, α, and T (Eq. (8)). Substituting these relations, the optimizing parameter can be rewritten as :

$$T = \frac{g_c I_{sp}^2}{2\eta\alpha}\left[\frac{2\lambda_{total}}{(1-\lambda_{total})\ln(1-\lambda_{total})^{-1}}-1\right] \tag{38}$$

Eqs. (37) and (38) are sufficient to solve the two-burn rendezvous problem. By choosing an initial value of λ_{total} and substituting into Eq. (38), an initial trip time can be found. (From Shepherd, an initial (maximum) λ_{total} value of 0.796812 is suggested. [4]) The trip time can then be substituted into Eq. (37) to find a new value of λ_{total}. Repeated iteration using the average of the λ_{total} values as the new λ_{total} was found to converge quite rapidly in most cases.

Once a converged solution is found, several dependent variables of interest can easily be calculated. The first burn propellant ratio and c/V_e can be calculated from Eqs. (36) and (8) respectively. The structure, payload, and initial & final thrust-to-weight ratios are then given by:

$$\frac{M_s}{M_i} = \left(\frac{M_s}{M_p}\right)\lambda_{total} = \left(\frac{c}{V_e}\right)^2 \lambda_{total} \tag{39}$$

$$\frac{M_{pay}}{M_i} = 1 - \left(\frac{M_p}{M_i}\right) - \left(\frac{M_s}{M_i}\right) = 1 - \lambda_{total} - \left(\frac{c}{V_e}\right)^2 \lambda_{total} \tag{40}$$

$$\frac{F}{W_i} = \frac{1}{g}\frac{c\lambda_{stage}}{t_1} = \frac{g_c}{g}\frac{I_{sp}}{T}\lambda_{stage}(2-\lambda_{stage}) \tag{41}$$

$$\frac{F}{W_f} = \frac{1}{g}\frac{-c\lambda_{stage2}}{(t_2-t_1)(1-\lambda_{stage2})} = -\frac{g_c}{g}\frac{I_{sp}}{T}\frac{\lambda_{stage}(2-\lambda_{stage})}{(1-\lambda_{stage})^2} \tag{42}$$

As in the single burn acceleration case, payload ratios can be specified and the limiting I_{sp}'s and α's can be calculated. The value of the parameter c/V_e for each corresponding value of payload ratio must be found first. This can be accomplished by solving the rocket equation (rewritten in terms of mass ratios) and the optimization parameter simultaneously after specifying the desired payload fraction. These equations are coupled, but were found to be easily solved by iteration. Shepherd had provided an approximate value of 0.505 for the zero payload case. Alternate and more exact values were desired for analysis purposes. For 0% and 25% payload fractions, the values of c/V_e were calculated to be 0.504976295 and 0.736886943 respectively. The value of λ_{stage} can then be found by inserting the optimization parameter Eq. (11) into the rewritten rocket equation Eq. (10) (replacing the exponential term), substituting Eq. (36) for the equivalent $\Delta V/c$ term, and solving for λ_{stage}. The result is:

$$\lambda_{stage_{Opt}} = 1 - \exp\left[\frac{\left[\left(\frac{M_{pay}}{M_i}\right) - 1\right]\left(\frac{c}{V_e}\right)^2}{\left[\left(\frac{c}{V_e}\right)^2 + 1\right]\left[\left(\frac{c}{V_e}\right)^2 + \left(\frac{M_{pay}}{M_i}\right)\right]}\right] \tag{43}$$

Once λ_{max} is known, the optimum travel time can be easily calculated from Eq. (34). The optimum α can then be calculated from Eq. (8). At this point, the two-burn problem and the parameters of interest are solved. These relations can be manipulated by hand or easily assembled into a short computer routine.

Derivation of the Solution to the Four-Burn Round Trip Problem

A similar set of equations were derived for a four-burn round trip mission, where the spacecraft would carry all of its propellant without the use of in situ propellant refueling. The equations were found to be similar to the two-burn rendezvous mission. The two-burn rendezvous mission can be thought of as the return leg of a round trip mission. The velocity equations are of the same form as the two-burn mission, as are the acceleration equations. Since the outgoing and returning distances will be assumed to be equal

$$S_{outgoing} = S_{returning} \qquad \frac{g_c I_{sp} T_{outgoing} \lambda_1}{2 - \lambda_1} = \frac{g_c I_{sp} T_{returning} \lambda_3}{2 - \lambda_3} \tag{44}$$

$$thus: \quad \frac{T_{outgoing}}{T_{return}} = \frac{\lambda_3 (2 - \lambda_1)}{\lambda_1 (2 - \lambda_3)} \tag{45}$$

The relationships between trip times and λ's must be found. Since the mass flow rates are constant:

$$\frac{T_{outgoing}}{T_{return}} = \frac{\dfrac{M_{p_1} + M_{p_2}}{M_i}}{\dfrac{M_{p_3} + M_{p_4}}{M_i}} = \frac{\lambda_1 + \lambda_1 (1 - \lambda_1)}{\lambda_3 (1 - \lambda_1)^2 + \lambda_3 (1 - \lambda_3)(1 - \lambda_1)^2} \tag{46}$$

Equating Eqs. (45) and (46), λ_1 and λ_3 are thus related by:

$$\lambda_3 = \frac{\lambda_1}{1 - \lambda_1} \tag{47}$$

An expression for λ's in terms of λ_{total} can be found since:

$$\lambda_{total} = \frac{M_{p_1}}{M_i} + \frac{M_{p_2}}{M_i} + \frac{M_{p_3}}{M_i} + \frac{M_{p_4}}{M_i} = \lambda_1 + \lambda_1 (1 - \lambda_1) + \lambda_3 (1 - \lambda_1)^2 + \lambda_3 (1 - \lambda_3)(1 - \lambda_1)^2 \tag{48}$$

Substituting Eq. (47) into Eq. (48) results in a quadratic expression for λ_1 in terms of λ_{total}:

$$\lambda_1^2 - \lambda_1 + \frac{\lambda_{total}}{4} = 0 \quad \textit{with roots of:} \quad \lambda_1 = \frac{1 \pm \sqrt{1-\lambda_{total}}}{2} \tag{49}$$

Only the negative radical term is meaningful. Notice that this first burn propellant fraction is half the expression found in the two-burn rendezvous case (Eq. (36)).

Finally, the outgoing trip time can be expressed as a fraction of the total round trip time by again noting that mass flow rate is constant:

$$\frac{T_{outgoing}}{T} = \frac{\dfrac{M_{p_1}+M_{p_2}}{M_i}}{\dfrac{M_{p_1}+M_{p_2}+M_{p_3}+M_{p_4}}{M_i}} = \frac{\lambda_1(2-\lambda_1)}{\lambda_1(2-\lambda_1)+\lambda_1(2-3\lambda_1)} \tag{50}$$

Substituting Eq. (50) into the round trip distance Eq. (44):

$$S_{roundtrip} = 2S_{rendezvous} = 2g_c I_{sp} T_{outgoing}\left(\frac{\lambda_1}{2-\lambda_1}\right) = \frac{g_c I_{sp} T}{2}\left(\frac{\lambda_1}{1-\lambda_1}\right) \tag{51}$$

As was done for the rendezvous case, this can be solved for λ_{total} using Eq. (49). For convenience, the distance can be written in terms of $S_{rendezvous}$ rather than $S_{roundtrip}$:

$$2S_{rendezvous} = \frac{g_c I_{sp} T\left(1-\sqrt{1-\lambda_{total}}\right)}{2\left(1+\sqrt{1-\lambda_{total}}\right)} \quad or \quad \lambda_{total} = 1-\left(1-\frac{8S_{rendezvous}}{g_c I_{sp} T + 4S_{rendezvous}}\right)^2 \tag{52}$$

Using this equation and the optimization parameter Eq. (38), the round trip mission trip time can be calculated in the same way as was done for the rendezvous case. The c/V_e, structure, and payload ratios are calculated the same as in the rendezvous mission (Eqs. (8), (39), and (40)). The optimum fixed payload (Eq. (43)) is reduced by a factor of one half as per Eq (49). The thrust-to-weight ratios are:

$$\frac{F}{W_i} = \frac{1}{g}\frac{c\lambda_{stage}}{t_1} = 4\frac{g_c}{g}\frac{I_{sp}}{T}\lambda_{stage}\left(1-\lambda_{stage}\right) \tag{53}$$

$$\frac{F}{W_f} = \frac{1}{g}\frac{-c\lambda_{stage4}}{(t_4-t_3)(1-\lambda_{stage4})} = -4\frac{g_c}{g}\frac{I_{sp}}{T}\frac{\lambda_{stage}\left(1-\lambda_{stage}\right)}{(1-2\lambda_{stage})^2} \tag{54}$$

RESULTS

Iterations on the trip time/optimization parameter equations were well behaved and converged rapidly. Although sample hand calculations were not found to be overly burdensome, a simple computer routine was written to calculate almost all of the following data. As an example, rendezvous and round trip missions to Jupiter were calculated. After this data was generated, the same mission-space parameters were calculated using a large, numerical integration trajectory optimization routine to independently verify the data and establish the limits of validity of this approach. The position of Jupiter was assumed to be at minimum distance from Earth (4.203 AU) at departure. Round trip missions assumed no dwell time at the destination and no significant relative motion of the planets. Ranges of I_{sp} from 50,000 to 200,000 seconds and α from 100 to 1000 kW/kg were representative of what a propulsion system must operate at to perform multi-week/ multi-month transfers to this nearest of major planets. Overall propulsion system efficiency was set to unity. Figure 3 illustrates rendezvous mission trip time as a function of both I_{sp} and α. A rendezvous trip time to Jupiter of six weeks, for example, will require propulsion technologies with I_{sp}'s of at least 50,000 seconds and α's of at least 100 kW/kg. For constant I_{sp}, increasing α always decreases trip time since the engine jet power-out increases (and/or the structure mass decreases). For constant α, increasing I_{sp} always lengthens trip time as the propulsion system begins to act more like a low thrust electric device. I_{sp}'s beyond the 50,000 to 100,000 seconds levels are excessive for interplanetary ΔV's unless a corresponding (significant) improvement in α also occurs. A propulsion system engineer, planning to conduct proof of concept experiments, can use these observations to readily trade-off work on one parameter for the other depending on the judged engineering difficulty. Propulsion technology research can thus be guided by the knowledge of the relative merit of pursuing greater thrust vs. greater power.

Using the fixed payload mass fraction equations, payload contours can be superimposed onto the I_{sp} & α plots as a means of measuring performance. Two performance levels were calculated, a 0% (fastest trip time) and a 25% payload mass fraction. Other desired payload mass fractions between these two values can be approximated by interpolation. Greater values should not be estimated in this way due to the non-linearity of the function as c/V_e increases. For trip times of approximately six weeks, the maximum payload mass fractions for I_{sp}'s between 50,000 and 100,000 seconds were 3% to 25% respectively. To halve trip times (three weeks), I_{sp}'s would have to be doubled and α's would have to be improved by an order of magnitude.

Figure 4 illustrates the results for round trip missions, using similar ranges of I_{sp} and α. In the round trip mission, the spacecraft departing from Earth carries all the fuel required to perform the entire mission. By observation, the data illustrates that (like terrestrial travel) refueling at a destination is a compelling way to operate. For example, a zero payload/ 50,000 second I_{sp} vehicle that refuels at a destination would take half the time (2 X 40 days) to perform the same mission as a same initial weight vehicle carrying all of its propellant from the start. This enhances the attractiveness of travel to the major outer planets due to their abundant supply of potential fuels and propellants such as hydrogen, deuterium, and helium-3. These materials are plentiful in both free molecular form in planetary atmospheres and in bound forms on the surfaces of their moons. Major planets would not only be destinations, but transportation nodes as well, supplying resources for fuels/propellants and human consumption.

Figure 5 and Table 1 contain additional rendezvous mission data: mass fractions, trip times, and thrust-to-weight data for I_{sp} = 50,000 seconds plotted as a function of α. For fixed I_{sp}, as α increases, the amount of propellant needed increases at the expense of payload fraction. The structure ratio reaches a maximum at approximately 26% as Shepherd's work had predicted [4]. As trip time is reduced by increasing α, payload ratio becomes vanishingly small. Thrust-to-weight ratios depart from near constant values as α increases. Figure 6 and Table 2 contain similar data for round trip missions, plotted as a function of I_{sp}.

Example: Mission to Jupiter (Rendezvous)

Trip Time vs. Isp & Alpha

(Maximized payload ratio, minimized travel time)

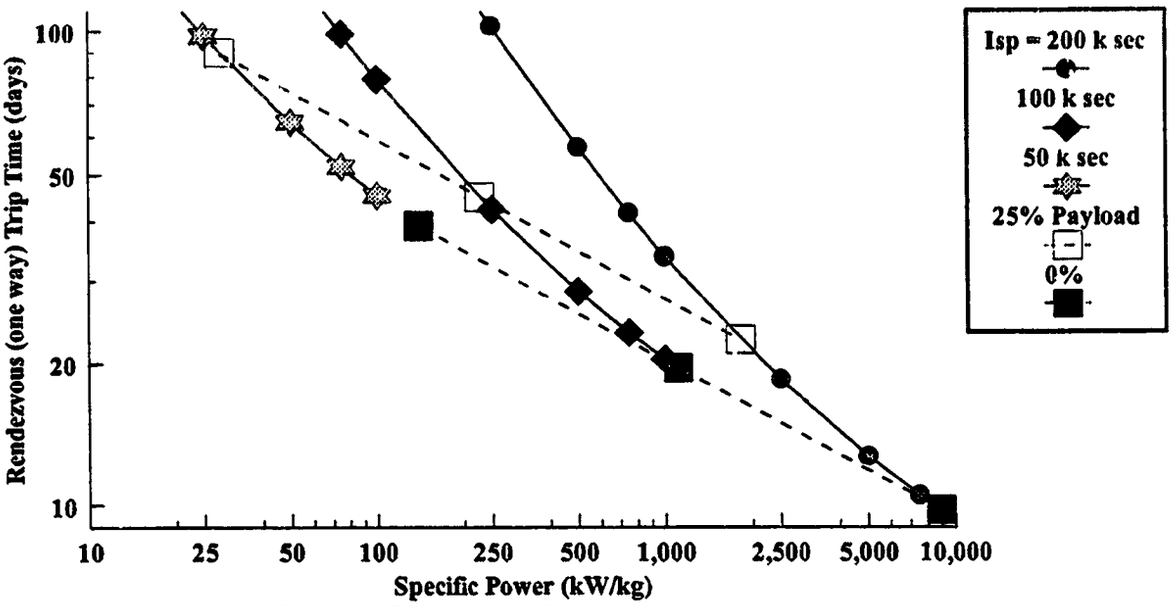

Figure 3. Rendezvous Mission TripTime

Example: Mission to Jupiter (Round Trip)

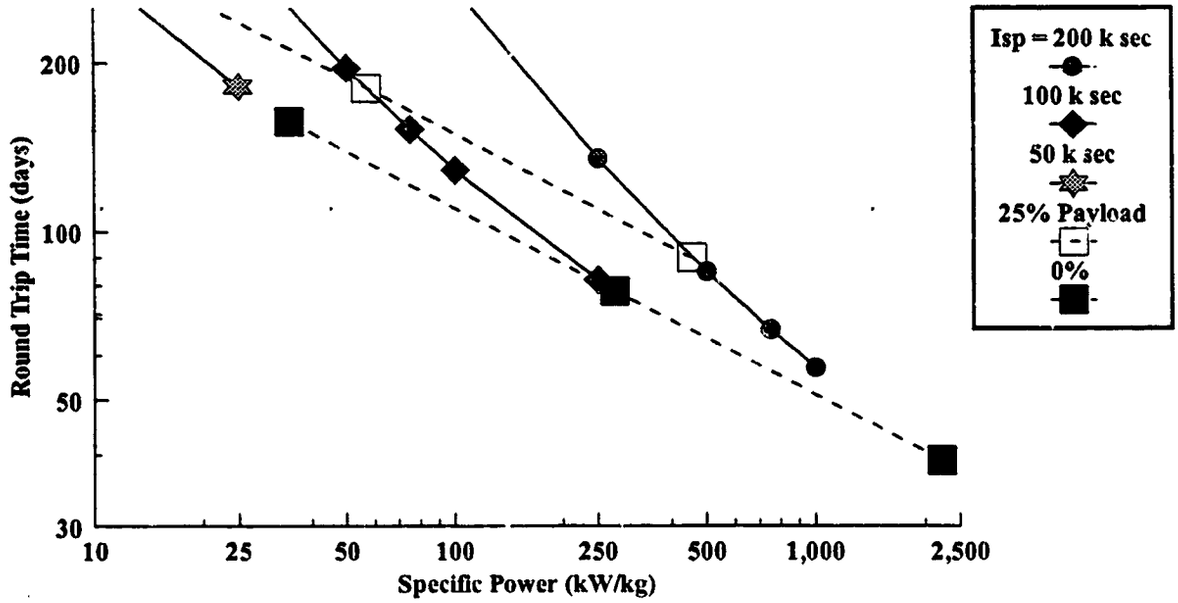

Figure 4. Round Trip Mission TripTime

Figure 5. Selected Mission Data

Example: Mission to Jupiter (Rendezvous)

Mass Ratio , Trip Time, & Thrust -to-Weight vs. Specific Power

Isp = 50,000 Sec

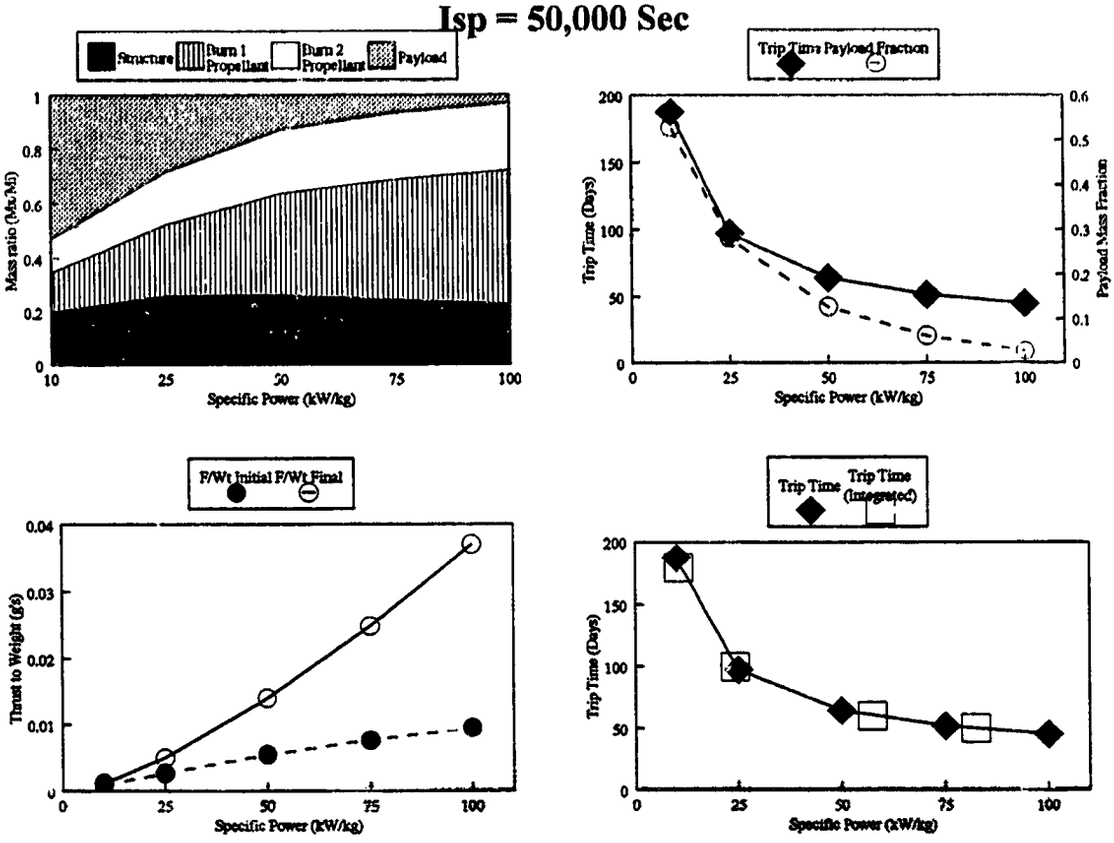

Table 1. Selected Mission Data

Isp = 50,000 Sec

Specific Power (kW/kg)	Time (days)	Payload Ratio	Structure Ratio	Total Prop Ratio	1st Burn Prop Ratio	c/Vc	F/Wt Initial Accel(g's)	F/Wt Final Decel(g's)
10	187.9	0.528	0.200	0.271	0.146	0.860	0.835E-3	0.115E-2
25	97.7	0.282	0.260	0.458	0.264	0.754	0.271E-2	0.500E-2
50	64.4	0.128	0.263	0.609	0.375	0.657	0.548E-2	0.140E-1
75	52.0	0.063	0.246	0.691	0.444	0.597	0.769E-2	0.249E-1
100	45.3	0.028	0.228	0.744	0.494	0.554	0.950E-2	0.371E-1

Alpha = 100 kW/kg

Specific Impulse (Sec)	Time (days)	Payload Ratio	Structure Ratio	Total Prop Ratio	1st Burn Prop Ratio	c/Vc	F/Wt Initial Accel(g's)	F/Wt Final Decel(g's)
50,000	45.3	0.028	0.228	0.744	0.494	0.554	0.950E-2	0.371E-1
100,000	79.3	0.468	0.219	0.313	0.171	0.837	0.457E-2	0.665E-2
150,000	143.1	0.758	0.113	0.129	0.067	0.934	0.157E-2	0.180E-2
200,000	236.5	0.882	0.057	0.061	0.031	0.969	0.595E-3	0.634E-3

Figure 6. Selected Mission Data

Example: Mission to Jupiter (Round Trip)

Mass Ratio , Trip Time, & Thrust -to-Weight vs. Specific Impulse

Alpha = 25 kW/kg

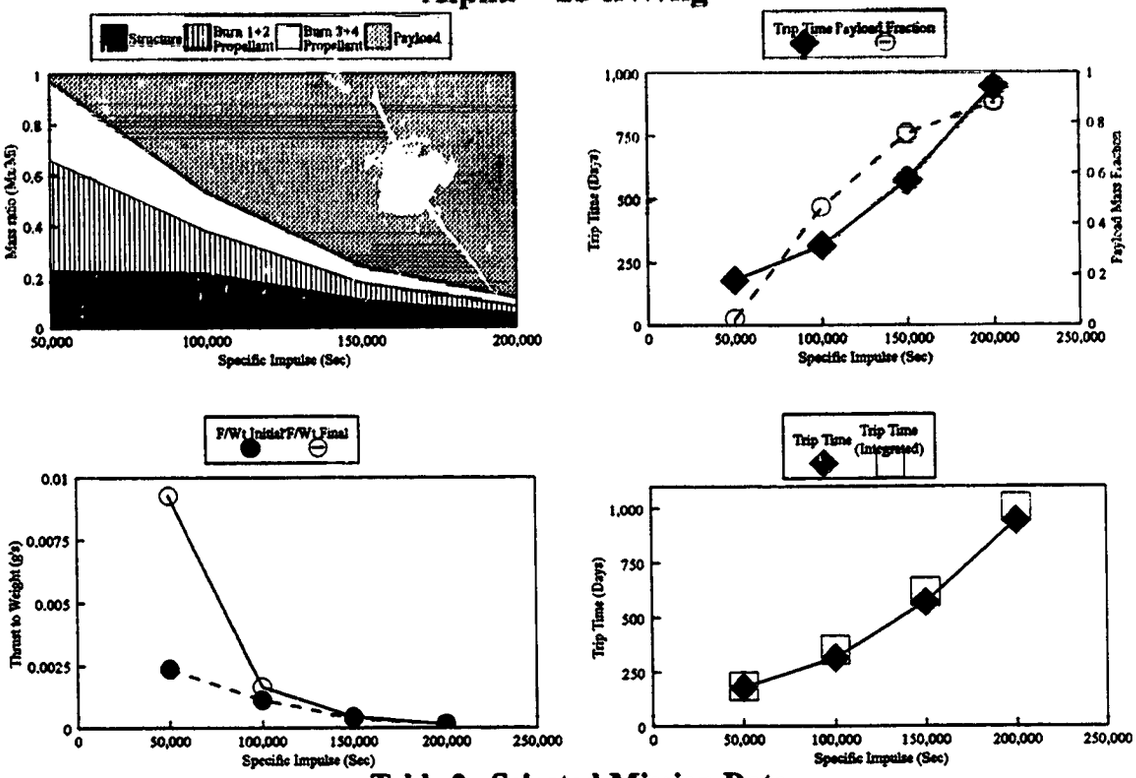

Table 2. Selected Mission Data

Isp = 200,000 Sec

Specific Power (kW/kg)	Time (days)	Payload Ratio	Structure Ratio	Total Prop Ratio	1st Burn Prop Ratio	c/Vc	F/Wt Initial Accel(g's)	F/Wt Final Decel(g's)
25	946.1	0.882	0.057	0.061	0.015	0.969	0.149E-3	0.158E-3
50	499.5	0.788	0.100	0.112	0.029	0.943	0.520E-3	0.585E-3
75	349.7	0.711	0.132	0.156	0.041	0.920	0.103E-2	0.123E-2
100	274.2	0.647	0.158	0.195	0.051	0.900	0.165E-2	0.204E-2
250	134.8	0.407	0.236	0.357	0.099	0.812	0.614E-2	0.956E-2
500	84.8	0.228	0.266	0.507	0.149	0.724	0.138E-1	0.280E-1
750	66.7	0.140	0.264	0.595	0.182	0.666	0.207E-1	0.510E-1
1000	57.1	0.090	0.255	0.655	0.206	0.624	0.266E-1	0.771E-1

Alpha = 25 kW/kg

Specific Impulse (Sec)	Time (days)	Payload Ratio	Structure Ratio	Total Prop Ratio	1st Burn Prop Ratio	c/Vc	F/Wt Initial Accel(g's)	F/Wt Final Decel(g's)
50,000	181.1	0.028	0.228	0.744	0.247	0.554	0.238E-2	0.927E-2
100,000	317.2	0.468	0.219	0.313	0.086	0.837	0.114E-2	0.166E-2
150,000	572.6	0.758	0.113	0.129	0.033	0.934	0.392E-3	0.450E-3
200,000	946.1	0.882	0.057	0.061	0.015	0.969	0.149E-3	0.158E-3

Table 3. Equation Summary

	1 Burn Flyby	2 Burn Rendezvous	4 Burn Round Trip
Total Distance	(13)	(35)	(52)
Optimized Trip Time	(16)	(37) & (38)	(52) & (38)

	Position	Velocity	Acceleration
Acceleration Phase	(12)	(20)	(25)
Deceleration Phase	(24)	(21)	(26)

To check the validity of these results, a series of integrated computer runs were performed using an existing, high fidelity interplanetary trajectory optimization routine [5]. A few of the results of these runs are superimposed onto the data generated and are shown in the last pl Figures 5 and 6. It was found that at these high power levels and with accelerations well in excess of the local acceleration due to solar gravity, trip time results agreed within 1% for rendezvous and 6% for round trip missions. The larger error in round trip times was noticeable in cases where travel times where a considerable fraction (or greater) of one Earth year. In these cases, the integration program significantly altered the outgoing trip to accommodate the change in relative position of the Earth upon return. Since the approach outlined in this paper does not account for change in the relative positions of the planets implicitly, closer agreement for round trip missions is limited to the time scales similar to those of rendezvous missions. Table 3 provides an overall summary of pertinent trip time relations and equations of motion for the analyst.

CONCLUSIONS

A simple, analytic approximation was shown to provide a means to readily calculate trip time and performance of propulsion systems of very high I_{sp} & α for human interplanetary space missions. Simultaneous solution of two equations was shown to be all that was necessary without having to resort to complex computer programs to integrate trajectories. The simplifying assumptions of one dimensional motion, field-free space, constant thrust/I_{sp}/α, and near straight line (radial) trajectories between the planets permitted data of sufficient accuracy to be generated, providing insight into the relationships between distance, I_{sp}, and α. An optimizing parameter was included to maximize performance and minimize trip time. Example data was provided for rendezvous and round trip missions to Jupiter, illustrating trip times and payload mass fractions for a wide range of I_{sp} and α. Results indicated that a propulsion system technology must be capable of I_{sp}'s of at least 50,000 seconds and α's of at least 100 kW/kg to travel to Jupiter in 6 weeks (rendezvous) and still allow for a 3% payload mass fraction. Overall results were shown to agree within 1% and 6% (rendezvous and round trip, respectively) of data independently generated, verifying that for preliminary analysis, a technology planner need not resort to large, high fidelity computer programs that require expert operators in order to accurately characterize the propulsion system parameter-space. Comparison of data between two-burn rendezvous and four-burn round trip missions illustrated the extreme attractiveness of in situ refueling at the destination planet. Few propulsion technologies have been identified that are expected to be capable of providing such demanding I_{sp}'s and α's. Inertial confinement fusion systems may be capable of I_{sp}'s between 50,000 to 270,000 seconds and α's as large as 100 kW/kg [6]. Antiproton-catalyzed fusion and antiproton annihilation are other potential space propulsion concepts that are expected to operate at or beyond this regime.

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Discussions with Dr. Stanley Borowski of the NASA Lewis Research Center and Professor Terry Kammash of the University of Michigan were invaluable in the formulation of this approach. Their earlier work deriving selected equations using an alternate approach to this problem provided the necessary guidance and confirmation of the validity of this method [7,8]. Special thanks to Leon Gefert and Leonard Dudzinski of the NASA Lewis Research Center for the numerous integrated trajectories that were run to validate the equations.

NOTATION

A	acceleration (ft/sec**2)
α	specific power (ft lbf/(sec lbm))
c	exhaust velocity (ft/sec) = $g_c I_{sp}$
F	thrust (lbf)
g	gravitational acceleration: 32.1739 (ft/sec**2)
g_c	conversion constant: 32.1739 (lbm/lbf)(ft/sec**2)
I_{sp}	specific impulse (lbf sec/lbm) = thrust per mass flow rate
λ	propellant mass fraction (M_p/M_i)
M	mass (lbm)
η	overall propulsion system efficiency
P	jet power (ft lbf/sec)
S	distance (ft)
t	time (sec)
T	trip time (sec)
V_e	characteristic velocity (ft/sec)
V	velocity (ft/sec)
ΔV	velocity increment (ft/sec)
W	weight (lbf)

Subscripts

0,1,2,3,4	propulsion system burns & phases of flight
f	final
i	initial
p	propellant
pay	useful payload
s	structure (including tankage & power supply)

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A simple, analytic approximation is derived to calculate trip time and performance for propulsion systems of very high specific impulse (50,000 to 200,000 seconds) and very high specific power (10 to 1000 kW/kg) for human interplanetary space missions. The approach assumed field-free space, constant thrust/constant specific power, and near straight line (radial) trajectories between the planets. Closed form, one dimensional equations of motion for two-burn rendezvous and four-burn round trip missions are derived as a function of specific impulse, specific power, and propellant mass ratio. The equations are coupled to an optimizing parameter that maximizes performance and minimizes trip time. Data generated for hypothetical one-way and round trip human missions to Jupiter were found to be within 1% and 6% accuracy of integrated solutions respectively; verifying that for these systems, credible analysis does not require computationally intensive numerical techniques.

14. SUBJECT TERMS

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